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Cahier N° 04.01.44

15 janvier 2004

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The Competitive Firm under both Input and output Price Uncertainties with Futures Markets and Basis Risk

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January 2004

Abstract:We study the case of a competitive firm exposed to both input and output price risk. In an expected utility framework, we elicit the separation theorem, and show the positive impact of derivatives markets on the optimal output. We also show that in the case of several inputs, the risk-averse firm accord a certainty premium to the most certain input.

Key Words:uncertainty,risk aversion, prudence, futures markets

JEL Classification:D81,D21

Preliminary Version

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[†]Thanks are due to Jacques Percebois for his contribution in the preparation of this version. The usual caveat of course applies.

1 Introduction

1.1 Production decision under uncertainty and hedging: a survey

"Theory of the Competitive Firm under price Uncertainty" is in the common sense due to Sandmo (1971). Even if his principal result had always been highlighted by Baron (1970) and Rotschild and Stiglitz (1970), we have Sandmo to thank for his more complete and clear presentation. His principle result describes consequences of output price uncertainty on the behavior of a risk-averse expected utility maximizer firm. Compared to either deterministic or risk-neutrality cases, risk aversion leads to diminish output quantity. The main contribution of Sandmo was to establish the relationship between production theory and the recent Arrow-Pratt notion of risk aversion.

These results were discussed in the 1970s. Mainly, Batra and Ullah (1974) viewed the input side and Ishii (1977) completed the Sandmo's paper by envisaging a change in risk aversion of the agent. None of these models allude to derivatives markets and hedging possibilities. This gap was filled by Holthausen (1979) and Feder, Just and Schmitz (1980). Their main result, the *separation property* is prominent in the financial literature. Roughly speaking, this property states that when a forward market is available for a risk-averse agent, this agent will consider the current forward price exactly as a certain future spot price and then equalize his marginal production cost to this price. Its production decision become in that case independent of its subjective price distribution and moreover of its attitude towards risk[1]. The forward price is then the benchmark variable for firm's production.

The main following contributions are summarized in table (1). This table shows mainly that separation property applies when a forward market is available. Note that separation property fails when a independent unhedgeable background risk is introduced. It also fails when basis risk is taken into account.

1.2 Motivation of the paper

The aim here is to focus on the input price risk in an expected utility framework. As mentioned in the table, just two contributions took this point into account. First, Paroush and Wolf (1992) consider a deterministic two-factors production function and just one of the two inputs is viewed as risky. A futures market[2] exists for this input and basis risk is envisaged. Both production function and output price are deterministic. No simple result is derived from this analysis, but influences from multiple factors, as basis risk magnitude, sign of crossed-derivatives and market situation (*contango* or *normal backwardation*)[3] are studied.

Viaene and Zilcha (1998) consider a firm exposed to both input and output price uncertainties[4]. A futures market is available for the output but not for the input. Authors conclude to

[1]Note that this separation property is a bit similar to the Tobin's one (1958)in the case of portfolio choice with one risk-free asset.

[2]An exhaustive presentation of derivatives can be found in "Futures, Options and Derivatives", Hull, 2003.

[3](*contango* or *normal backwardation* concepts can also be found in Hull (2003) or in Simon and Lautier (2003).

[4]Risk and uncertainty refer to the same notion here and the difference between those two concepts in the

Authors	**Approach**	**Hypothesis**	**Conclusions**
Holthausen (1979)	EU	Output price Forward Markets	Separation $\nearrow$ *risk* $\Rightarrow$ $\nearrow$ *hedge* Bench = forward
Feder-Just Schmitz (1980)	EU	Output price	Separation Bench = forward
Paroush-Wolf (1986)	EU	Output price Forward et Futures	Forward CS Separation Bench = forward
Antonovitz Nelson (1988)	EU	Output price Forward et Futures	Forward CS Separation Bench = forward
Safra-Zilcha (1986)	Mon. Pref	Output Price Forward Markets	Separation
Paroush-Wolf (1992)	EU	Input Price (1/2) Futures	Separation fails for Basis Risk
Briys-Crouhy Schlesinger (1993)	EU	Output Price Background risk	NB: $N* < N** < Q$ CONT : $Q < N** < N*$
Wong (1996)	EU	Output Price Background risk	$\searrow Q$ if Kimball (93) $\searrow Q$ if DARA Ross(81) Separation fails
Adam-Müller (1997)	EU	Income Exchange rate Forward rate	Separation fails
Viaene-Zilcha (1998)	EU	Input Price No hedging	Separation fails

Table 1: **Hedging and Uncertainty Literature**

failure of separation property in the general case. Note that this approach is very close to the *background risk* approach.

The case where the firm has the possibility to manage its risk both on input and output side is remarkable and has not been studied yet. Liberalization has decreased the singularity of this situation. Lots of competitive markets and following derivatives products emerged due to the

Knight'sense (1921) are ignored.

deregulation. In the energy sector, for instance, lots of firms are now exposed to uncertainty before and after their production process. For an electricity producer, gas price is risky as well as electricity price is, and futures markets exist in both cases[5]. We can also think to an oil refinery (crude oil into gasoline or heating oil) or a aluminium producer (aluminium and electricity price risks).

The paper is organized as follows: second section considers the case of forward markets in the multi-risks framework, third one envisages two inputs and focuses on the demand variations due to the uncertainty. Section four to conclude.

2 Production and hedging decisions without basis risk

2.1 The model

The model is a two-periods model ($t = 0, 1$). Consider a firm producing a unique good y bought in the second period at a price $\tilde{p}$ [6] in the second period. In the first period (*ex ante*) the price $\tilde{p}$ is unknown by the firm. The output quantity is $F(x)$ with F a concave production function[7] with a single input x. F is a deterministic production function[8] or in other words quantity is perfectly known when x is chosen. In the same way, input price $\tilde{c}$ is unknown (*ex ante*) and is introduced linearly in the profit function (unit cost). Random aspect of $\tilde{p}$ and $\tilde{c}$ are formalized as follows:

$$\tilde{p} = \bar{p} + \gamma\epsilon \quad ; \quad \tilde{c} = \bar{c} + \delta\eta$$

where $\bar{p}$ and $\bar{c}$ are respective means and γ and δ are respective standard-deviations. ϵ and η are zero-mean and unit-variance noises. Moreover noises are assumed to be statistically independent that is to say $E(\epsilon\eta) = 0$.

Assume now that two futures markets are available, the first for y and the second for x. Futures prices in the first period for these two goods are respectively given by p_0 et c_0 and assume, in a first step, that basis risk is absent. Futures prices and spot prices in second period will then be equal, or:

$$\tilde{f}_p = \tilde{p} \quad et \quad \tilde{f}_c = \tilde{c}$$

Let g and h be quantities bought - or sold - on futures markets respectively for y and x, profit function of the firm can then be written:

$$\tilde{\Pi} = \tilde{p}.F(x) - \tilde{c}.x - g(\tilde{p} - p_0) + h(\tilde{c} - c_0) \tag{1}$$

[5]Concerning electricity, remember that markets are simply emerging markets(see EIA Report "Derivatives and Risk Management in the Petroleum, Natural Gas, and Electricity Industries", October 2002).

[6]Tilde always mention random variables.

[7]Marginal productivity will be noted F_1 and its first derivative F_{11}, thus $F_1 > 0$ and $F_{11} < 0$.

[8]Stochastic production function is generally assumed in the agricultural world.

In the same way as Holthausen (1979) and all models presented in the survey section, all control variables have to be chosen in the first period and because of the two-periods model, no time-inconsistency problem can arise. To summarize, in the first period the firm has to choose its output, and then its input demand, and its two hedge levels. All decisions are taken in an uncertainty framework because of both input and output price risks. In the second period the all output is sold at the competitive price.

Remark 1 *We implicitly assume a instantaneous production process in order to avoid price change possibilities during production, and consequently a third period to the model. Thus we do not take into account the time resolution of uncertainty problem described for instance by Eeckhoudt, Gollier and Schlesinger (2004)*[9]*.*

All decisions are taken for the firm by an individual decision-maker with a classical Von-Neumann Morgenstern utility function $u(.)$. Remember VNM properties: $[E(u(.))]' > 0$ and $[E(u(.))]'' < 0$ following identical properties for the original utility function $u(.)$. Decision-maker's problem can then be written:

$$\max_{x,g,h} E\big(u(\widetilde{\Pi})\big) \tag{2}$$

Due to the concavity of the utility function and neoclassical production function's properties, a unique interior solution to the problem is given by the following three first-order conditions:

$$\partial E\big(u(\tilde{\Pi})\big)/\partial x = E\big[u'(\tilde{\Pi}).[F_1(x).\tilde{p} - \tilde{c}]\big] = 0 \tag{3}$$

$$\partial E\big(u(\tilde{\Pi})\big)/\partial x = E\big[u'(\tilde{\Pi}).[p_0 - \tilde{p}]\big] = 0 \tag{4}$$

$$\partial E\big(u(\tilde{\Pi})\big)/\partial x = E\big[u'(\tilde{\Pi}).[\tilde{c} - c_0]\big] = 0 \tag{5}$$

Or by integrating expressions retained for $\tilde{p}$ and $\tilde{c}$:

$$[\bar{p}F_1(x) - \bar{c}].E\big(u'(\tilde{\Pi})\big) + \gamma F_1(x)E\big(u'(\tilde{\Pi}).\epsilon\big) - \delta E\big(u'(\tilde{\Pi}).\eta\big) = 0 \tag{6}$$

$$[p_0 - \bar{p}].E\big(u'(\tilde{\Pi})\big) - \gamma E\big(u'(\tilde{\Pi}).\epsilon\big) = 0 \tag{7}$$

$$[\bar{c} - c_0].E\big(u'(\tilde{\Pi})\big) - \delta E\big(u'(\tilde{\Pi}).\eta\big) = 0 \tag{8}$$

[9]"Optimal consumption and the timing of the resolution of uncertainty", forthcoming *European Economic Review*.

By summation of the three conditions - with a $F_1(x)$ coefficient for the second - we obtain:

$$[F_1(x).p_0 - c_0].E\big(u'(\tilde{\Pi})\big) = 0 \tag{9}$$

2.2 Solutions and Analysis

2.2.1 Production Decision

Because of the marginal utility strict positivity, equation (9) confirms the well-known separation theorem.

Proposition 1 (Separation) *Decision production is independent of its subjective distribution about input and output price risks. This decision is also independent of firm's attitude towards risk.*

Proof is straightforward because $F_1(x).p_0 - c_0$ is a non-stochastic term. This property had never been envisaged for the input and output price risks case. The model confirm the intuition that when perfect derivatives markets are available, production decision and hedge decision are separable. For the firm the interest is on decisions' timing. Once forward price is known, production capacity can be chosen without taking into account spot price evolution. Flexibility in production is not necessary due to the all-adjustment made in derivatives markets[10].

2.2.2 Hedging decision

Jointly to production decision, adjustment in derivatives markets for risk-aversion motive is achieved. On each market - input or output - decision depends on market situation. Results are summarized in the table (2). We can see that full-hedging paradigm is a very particular case, corresponding to the unbiasedness in both markets. In all other situations, there is either under or overhedge. Note that these results consider the two risks respectively on input and output price as independent.

2.3 Imperfections on forward markets

We consider imperfections on forward markets by looking the extreme case of market absence. Consider first that no efficient derivatives market exists for output. After evaluation, we show as Feder, Just and Schmitz (1980), the positive effect of forward markets on production. For a neoclassical (concave) production function, the firm choose a lower output because of its risk-aversion. By a lower output, we mean that a concrete difference exists between an agent who know the future spot price and an agent with an expected spot price equal to the certainty

[10]For more about production flexibility, see "Forward Markets and the Behavior of the Competitive Firm with Production Flexibility", *Bulletin of Economic Research*, Wong(2003).

output / input	*Normal Backwardation*	*Unbiased*	*Contango*
Normal Backwardation	$g < F(x); h > x$ Under output Over input	$g = F(x); h > x$ Full output Over input	$g > F(x); h > x$ **Double overhedge**
Unbiased	$g < F(x); h = x$ Under output Full input	$g = F(x); h = x$ **Double full hedge**	$g > F(x); h = x$ Over output Full input
Contango	$g < F(x); h < x$ **Double underhedge**	$g = F(x); h < x$ Full output Under input	$g > F(x); h < x$ Over output Under input

Table 2: **Input and Output Hedging Positions**

future spot price of the first agent. Note that this case is particularly close to Sandmo (1971) because of the total risk elimination on input risk due to input forward market availability.

In the reverse case, where an efficient derivatives market on output is available only, result is symmetrical, and decision production is lower too in the decreasing returns to scale case. The effect of introduction of a forward market is positive on production decision again[11] even if in particular case this issue has been discussed (multiplicity of exchange places for instance).

3 Risk premium in the multi-input case

3.1 The model

We now consider a firm with a unique output at a non-stochastic price p. The output $F(x_1, x_2)$ is made with two *substitutable* inputs with prices $\tilde{p}_1$ and $\tilde{p}_2$. Prices are not known *ex ante*. A *futures market* exists for each input. *Futures* prices are f_1^0 , f_2^0 and $\tilde{f}_1$, $\tilde{f}_2$ respectively in the first and the second period. If we call g and h quantities respectively bought for x_1 and x_2, profit can be written as follows:

$$\tilde{\Pi}(x_1, x_2, g, h) = pF(x_1, x_2) - \tilde{p}_1 x_1 - \tilde{p}_2 x_2 - g(f_1^0 - \tilde{f}_1) - h(f_2^0 - \tilde{f}_2) \tag{10}$$

There are 4 first order conditions :

$$\partial E\big(u(\tilde{\Pi})\big)/\partial x_1 \equiv v_1 = E\big[u'(\tilde{\Pi})(pF_1 - \tilde{p}_1)\big] = 0 \tag{11}$$

$$\partial E\big(u(\tilde{\Pi})\big)/\partial g \equiv v_2 = E\big[u'(\tilde{\Pi})(\tilde{f}_1 - f_1^0)\big] = 0 \tag{12}$$

[11] Note that with forward markets production decision under uncertainty is also independent from fixed costs (Cayatte, 2004).

$$\partial E\big(u(\tilde{\Pi})\big)/\partial x_2 \equiv v_3 = E\big[u'(\tilde{\Pi})(pF_2 - \tilde{p}_2)\big] = 0 \tag{13}$$

$$\partial E\big(u(\tilde{\Pi})\big)/\partial h \equiv v_4 = E\big[u'(\tilde{\Pi})(\tilde{f}_2 - f_2^0)\big] = 0 \tag{14}$$

As in the last sections, *spot* and *futures* prices are randomized by an additive noise. For the multiplicative case, see Adam-Müller (2002), Mahul(2002) Alghalith (2003) and Alghalith and Dalal (2003). Thus we have :

$$\tilde{p}_1 = \bar{p}_1 + \psi\alpha \quad ; \quad \tilde{f}_1 = \tilde{p}_1 + \beta\varphi = \bar{p}_1 + \psi\alpha + \beta\varphi$$

$$\tilde{p}_2 = \bar{p}_2 + \gamma\epsilon \quad ; \quad \tilde{f}_2 = \tilde{p}_2 + \delta\eta = \bar{p}_2 + \gamma\epsilon + \delta\eta$$

With $E(\alpha) = E(\varphi) = E(\epsilon) = E(\eta) = 0$ and $E(\alpha^2) = E(\varphi^2) = E(\epsilon^2) = E(\eta^2) = 0$.

Statistical independence between all noises are given by :

$E(\alpha\varphi) = E(\epsilon\varphi) = E(\eta\epsilon) = E(\alpha\eta) = E(\alpha\epsilon) = E(\eta\varphi) = 0$

F_1 and F_2 are the first derivatives respectively on the first and the second input, then FOC can be now written :

$$v_1 = (pF_1 - \bar{p}_1)E\big[u'(\tilde{\Pi})\big] - E\big[u'(\tilde{\Pi})\alpha\big] = 0 \tag{15}$$

$$v_2 = (\bar{p}_1 - f_1^0)E\big[u'(\tilde{\Pi})\big] + \psi E\big[u'(\tilde{\Pi})\alpha\big] + \beta E\big[u'(\tilde{\Pi})\varphi\big] = 0 \tag{16}$$

$$v_3 = (pF_2 - \bar{p}_2)E\big[u'(\tilde{\Pi})\big] - \gamma E\big[u'(\tilde{\Pi})\epsilon\big] = 0 \tag{17}$$

$$v_4 = (\bar{p}_2 - f_2^0)E\big[u'(\tilde{\Pi})\big] + \gamma E\big[u'(\tilde{\Pi})\epsilon\big] + \delta E\big[u'(\tilde{\Pi})\eta\big] = 0 \tag{18}$$

If we assume, at the first time, that basis risk is zero on both *futures* markets, or $\psi = \delta = 0$. By adding the two first FOC and the two seconds, we show that input demands (and then output level) are independent from attitude towards risk and from anticipations on price distribution.

Proposition 2 *With two or more stochastic-price inputs, and without basis risk, the separation property holds.*

Proof 1 *No basis risk means : $\psi = \delta = 0$. By adding (15) and (16) we have : $pF_1 - \bar{p}_1 = \bar{p}_1 - f_1^0$. Symmetrically : $pF_2 - \bar{p}_2 = \bar{p}_2 - f_2^0$ or $pF_1 = f_1^0$ et $pF_2 = f_2^0$* $\square$

Consider now a non degenerated and non zero basis risk for both *futures* markets. In order to simplify the FOC and to introduce the Arrow-Pratt risk aversion coefficient, we can write the expected profit as :

$$\bar{\Pi}(x_1, x_2, g, h) = pF(x_1, x_2) - \bar{p}_1 x_1 - \bar{p}_2 x_2 - g(f_1^0 - \bar{f}_1) - h(f_2^0 - \bar{f}_2) \tag{19}$$

Consider now that basis risks on the two markets are different from zero, or $\psi \neq 0$ and $\delta \neq 0$. In order to simplify the FOC and integrate the Arrow-Pratt coefficient, we use the expected profit expression:

$$\bar{\Pi}(x_1, x_2, g, h) = pF(x_1, x_2) - \bar{p}_1 x_1 - \bar{p}_2 x_2 - g(f_1^0 - \bar{f}_1) - h(f_2^0 - \bar{f}_2) \tag{20}$$

We can then measure difference between profit and expected profit:

$$\tilde{\Pi} - \bar{\Pi} = (\psi\alpha + \beta\varphi)g - \psi\alpha x_1 + (\gamma\epsilon + \delta\eta)h - \gamma\epsilon x_2 \tag{21}$$

A first order Taylor's expansion from u' around $\bar{\Pi}$ gives:

$$u'(\tilde{\Pi}) \approx u'(\bar{\Pi}) + (\tilde{\Pi} - \bar{\Pi})u''(\bar{\Pi}) \tag{22}$$

Remark 2 *A first order Taylor's expansion implicitly assume a CARA utility function with normality distributions for input prices, or further a quadratic utility function. This approach highlight limits of the mean-variance approach (for a survey see Wagener (2001), "Comparative statics under certainty: the case of mean-variance preferences", discussion paper, University od Siegen). Note that Paroush and Wolf's papers (1989,1992), and consequently this paper, suggest also an implicit mean-variance approach due to the only first order Taylor's expansion (see Alghalith(2001))*[12].

Five useful approximations follow:

$$E\big[u'(\tilde{\Pi})\big] \approx u'(\bar{\Pi})$$

$$E\big[u'(\tilde{\Pi})\alpha\big] \approx \psi(g - x_1)u''(\bar{\Pi}) \quad ; \quad E\big[u'(\tilde{\Pi})\varphi\big] \approx \beta g u''(\bar{\Pi})$$

$$E\big[u'(\tilde{\Pi})\epsilon\big] \approx \gamma(h - x_2)u''(\bar{\Pi}) \quad ; \quad E\big[u'(\tilde{\Pi})\eta\big] \approx \delta h u''(\bar{\Pi})$$

A division of equations (15) to (16) by $E\big[u'(\tilde{\Pi})\big]$, and using last approximations :

$$v_1 \approx pF_1 - \bar{p}_1 - \psi^2 r(x_1 - g) \tag{23}$$

[12] Another relevant paper is Tsiang (1972) for the relationship between mean-variance preferences and normality.

$$v_2 \approx \bar{p}_1 - f_1^0 + \psi^2 r(x_1 - g) - \beta^2 rg \tag{24}$$

$$v_3 \approx pF_2 - \bar{p}_2 - \gamma^2 r(x_2 - h) \tag{25}$$

$$v_4 \approx \bar{p}_2 - f_2^0 + \gamma^2 r(x_2 - h) - \delta^2 rh \tag{26}$$

with r the Arrow-Pratt coefficient evaluated in $\bar{\Pi}$.

(23) to (26) equations allow to describe firm's behavior for production and hedging. None the less, these conditions are just approximations, and thus *prudence* concept, measurable by third derivative is neglected[13]. Consequences of *prudence* behavior will not be taken into account in this paper.

3.2 Solutions

3.2.1 Input demand and production

Equations (23) to (26) allows to calculate ratio between marginal productivity for each input. In the homogeneous first degree function case, this ratio can be expressed as an increasing function of the two inputs ratio itself[14], or $\frac{F_2}{F_1} = T(x_1/x_2)$. Consequently, we can by reciprocity express the two inputs ratio as an increasing function, so:

$$\frac{x_1}{x_2} = T^{-1}\Big[\frac{f_2^0 + \delta^2 rh}{f_1^0 + \beta^2 rg}\Big] \tag{27}$$

Proposition 3 *In the $\bar{\Pi}$ neighborhood, an increase in x_1 basis risk leads to a fall in the x_2 demand, compared to x_1 demand. The more [less] is the basis risk for x_2, the much more [less] is the fall. Similarly, a decrease in x_1 basis risk leads to an increase in the x_1 demand, compared to x_2 demand. The more [less] is the basis risk for x_2, the much more [less] is the increase. Symmetric properties follow for x_2.*

This last result allows to highlight the *risk premium* concept in the *theory of the competitive firm under price uncertainty.* When the firm is confronted to a choice between two (or more) inputs, its risk-aversion behavior leads to prioritize the most certain input (statistically speaking). When the idiosyncratic risk of the input can be hedged by a futures market, the basis risk subsists. The firm is henceforth induced to favour the input with the least basis risk (last unhedgeable risk).

[13]The original article is due to Kimball(1990), who extend precautionary saving highlighted by Leland(1968), Sandmo(1970) and Drèze and Modigliani(1972) in the consumption framework. For a global presentation see Gollier(2001).

[14]This is also the case for an homothetic function. Note that this is a realistic case for energy activity for instance.

As Sandmo (1971) and others, we show here consequences of the risk aversion in the production decision domain. Following Paroush and Wolf (1992) our interest is here on input price uncertainty. We show that risk-averse agent is ready to concede a part of his expected profit by reducing his demand for too risky input. In a expected profit approach, these decisions can appear non-optimal, but as in portfolio theory or insurance theory, they satisfy risk-averse behavior.

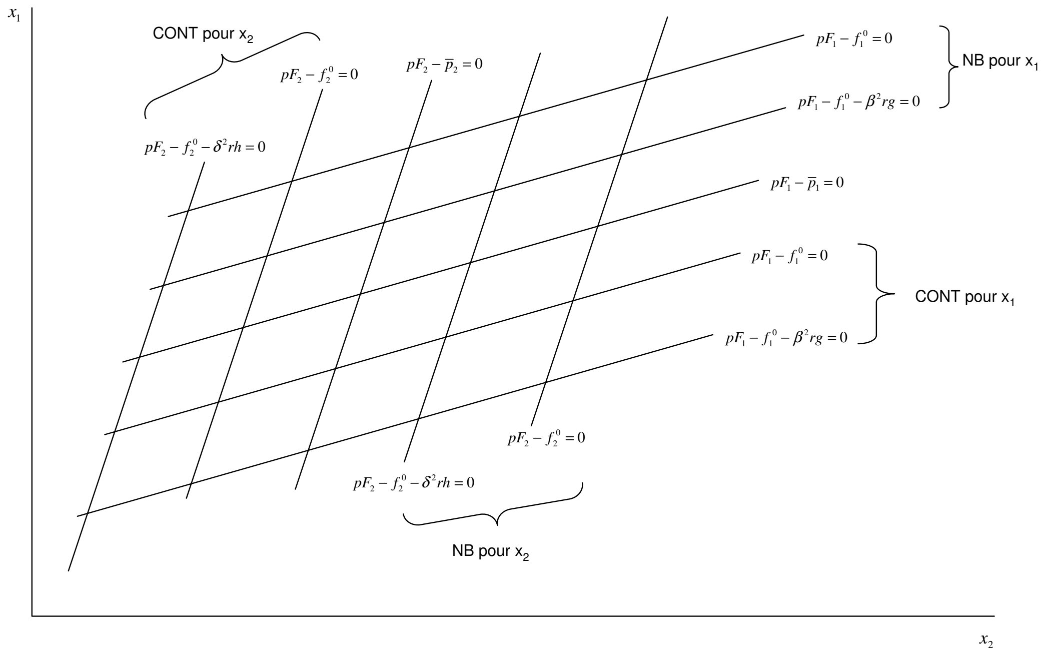

Figure 1: Input demands with basis risk - The substitutable factors case

Proposition 4 (Semi-separation) *Production decision is independent of the subjective input prices distributions. However, production decision is dependent of agent's attitude towards risk and subjective futures prices distributions. We can then conclude to semi-separation.*

This proposition explains how, to a large extent, price risk is eclipsed by basis risk. This is a crucial result for risk management for two reasons. First, it allows to be focused just on one risk, instead of two. Secondly, informed agents generally have a better skill on the basis risk management. Further more, basis risk variability is often below the physical or futures market variability level, as mentioned by Haushalter (2000)[15].

3.2.2 Hedging Decision

Equations (24) and (26) are symmetrical. They allow to establish optimal hedging level for each input. A simple transformation gives the hedge ratio, or the proportion of the physical

[15] G.D.Haushalter "Finance Policy, basis risk, and Corporate hedging: Evidence from Oil and Gas Producers", Journal of Finance, 55(1).

position which tends to be hedged:

$$x_1 - g = \frac{f_1^0 - \bar{p}_1}{\psi^2 r} + h\Big(\frac{\beta^2}{\psi^2}\Big) \tag{28}$$

$$x_2 - h = \frac{f_2^0 - \bar{p}_2}{\gamma^2 r} + h\Big(\frac{\delta^2}{\gamma^2}\Big) \tag{29}$$

Proposition 5 *If noises on all markets are independent, optimal hedge for each input is determined separately.*

Corollary 1 (Paroush-Wolf, 1992) *When a basis risk is added, a over-hedge could be preferred in normal backwardation.*

These two results follow from equations (28) and (29). If the proposition is straightforward, the corollary can be elicited. We can remark that if basis risk is sufficiently low, even if $f_i^0 < \bar{p}_i$[16], then we could indeed encounter a over-hedge situation. Remember that paradoxically, this situation is generally associated with the *contango* case. This property is close to the Paroush and Wolf's result (1992)[17].

4 Concluding remarks

We tried, in this paper, to have a look at the input price risk. Very often neglected because of the prominence of the output price in the profit, the input price question is now particularly relevant in the deregulation framework. Uncertainty spread through the all economic process and uncertainty sources are now multiple. Concerning input price uncertainty, we have here generalized several results generally associated with the output price risk. So what are the possible extensions?

In a first approach, input price problem seems isomorphic to the output price problem. Further results need to confirm this point, and perhaps a discussion about production function characteristics are conceivable.

Moreover, in the two-risks case when futures markets are available, a relationship is often existing between the two price evolutions. Nobody would sustain that gas and electricity prices evolve independently and lots of econometric studies confirm this assertion (see for instance "Relationship between Electricity and Natural Gas Futures Prices", *Journal of Futures Markets*, Emery-Liu (2002)).

[16] A normal backwardation situation.

[17] Note that Paroush and Wolf's result is not robust when basis risk tends to zero. In this case and in a contango framework, a under-hedge decision is optimal. This last result is of course an abberation if we refer to the hedging theory. An possible explanation would be perhaps limits due to the only first degree Taylor's development.

Finally, it would be relevant also to explicit analytically the risk premium mentioned in the last section in the general case, we mean without any restrictions on production function. It could allow to measure exactly the way to favour one input instead of an other because of risk considerations.

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